Technical Notes

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Mathematical Criterion for Unsteady Boundary-Layer Separation

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Introduction

RECENTLY, Wang and Shen[1] have observed that there is a certain coefficient (which they designate as k) which arises in the semi-similar formulation of the unsteady boundary layer problem and which apparently vanishes at the unsteady separation point. On the basis of this observation, Wang and Shen have conjectured that the vanishing of this coefficient indicates a critical point which "might actually be a barrier of the solution, indicating separation."[2] The purpose of this Note is to point out that there are additional solutions available which indicate a link between the vanishing of this coefficient and the separation point in some flows. It is also pointed out, however, that there are flows in which the coefficient vanishes but in which unsteady separation is neither expected nor indicated.

Analysis

The problem at hand is that of investigating incompressible, two-dimensional, unsteady boundary-layer flows using the method of semi-similar solutions. The appropriate boundary-layer equations and boundary conditions are well known and will not be repeated here. In the method of semi-similar solutions, the number of independent variables is reduced from three (x,y,t) to two (ξ,η) by an appropriate scaling. To this end we introduce new independent variables η and ξ and a dimensionless stream function f defined by

$$\eta = y/\nu^{1/2} g(x,t), \qquad \xi = \xi(x,t),$$
$$\psi(x,y,t) = u_\delta(x,t)\, g(x_g t)\, \nu^{1/2} f(\xi,\eta)$$

The continuity equation is satisfied identically by the introduction of a stream function and the momentum equation becomes, in terms of the new variables

$$\frac{\partial^2 f'}{\partial \eta^2} + \alpha_1 \frac{\partial f'}{\partial \eta} + \alpha_2 f' + \alpha_3 = \alpha_4 \frac{\partial f'}{\partial \xi}, \qquad f' = \frac{\partial f}{\partial \eta} = \frac{u}{u_\delta} \tag{1}$$

where

$$\alpha_1 = \left(g^2 \frac{\partial u_\delta}{\partial x} + u_\delta \frac{\partial g^2}{\partial x} \Big/ 2\right) f + \frac{\partial g^2}{\partial t}\eta/2 + u_\delta g^2 \frac{\partial \xi}{\partial x}\frac{\partial f}{\partial \xi}$$

$$\alpha_2 = -g^2 \frac{\partial u_\delta}{\partial x}\frac{\partial f}{\partial \eta} - g^2 \frac{\partial u_\delta}{\partial t} \Big/ u_\delta$$

$$\alpha_3 = g^2 \frac{\partial u_\delta}{\partial t} \Big/ u_\delta + g^2 \frac{\partial u_\delta}{\partial x}$$

$$\alpha_4 = g^2 \left(\frac{\partial \xi}{\partial t} + u_\delta \frac{\partial \xi}{\partial x} f'\right) = k$$

This form, in which f' is treated as the dependent variable, emphasizes the parabolic nature of Eq. (1).[3] The formulation of the semi-similar boundary layer equation employed here is identical to that of Williams and Johnson[4] and additional details of the semi-similar formulation of the unsteady problem may be found in Ref. 4. Wang and Shen employ a semi-similar formulation in which the independent variables are the velocity components, rather than the stream function. The difference is simply a matter of choice and not of substance. It is only important to note that the coefficient α_4 in the present formulation is identical with the coefficient k in the formulation of Wang and Shen[1] and Shen.[2]

Wang and Shen have observed that in the solutions obtained in Ref. 4, α_4 becomes zero at the unsteady separation point. On the basis of this observation, they have suggested that a "unified separation criterion for both the steady and unsteady cases might be possible if formulated in terms of k" and that this criterion is "$k=0$ at some interior point of the fluid." This suggestion was based entirely on the numerical solutions presented in Ref. 4. There are, however, additional semi-similar solutions which can be used to shed light on the above suggestion. In the present work we will investigate the behavior of the coefficient α_4, not only for the flows presented in Ref. 4 but also for two additional flows. The unsteady velocity at the edge of the boundary layer and the scaling parameters for the corresponding semi-similar solution for those flows are given in Table 1.

Results

The first case is the case presented in Ref. 4. The flow involved is an unsteady variation of the classical Howarth linearly retarded flow. In this case the solutions of Ref. 4 have been recalculated and new solutions have been obtained for values of the unsteadiness parameter, λ, of 1.5 and 2. These solutions are used to determine the variation of the coefficient α_4. The second case is an unsteady variation of the Falkner Skan flows. Solutions for this class of flows have been presented in Ref. 5. As in Ref. 4, the solutions in Ref. 5 indicate that the separation point is characterized by the simultaneous vanishing of the shear and the velocity, at a point within the boundary layer, in a coordinate system moving with the separation point. This is the Moore-Rott-Sears criterion for unsteady separation. Again, these solutions have been recalculated and some new solutions calculated to determine the variation of the coefficient α_4. Solutions have been obtained for M equal to -0.05 and λ equal to 0.25, 0.4, 0.5, and 1.0.

Case III represents the classical problem of the semi-infinite flat plate which is impulsively set into motion. The semi-similar scaling employed here is one developed by the author and one of his colleagues in connection with a study of wedge-type flows impulsively set into motion.[6] This particular scaling is one of several which might be used. It has, however, the advantages of 1) transforming an infinite region of integration $(0<x<\infty,\ 0\le t\le\infty)$ into a finite region $(0\le\xi\le 1)$ and 2) providing a reduced momentum equation which includes both the limiting cases appropriate to the problem; the

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Table 1 Three flows with semi-similar solutions

Case	$u_\delta(x,t)$	$g^2(x,t)$	$\xi(x,t)$	Ref.
I	$1-x/(1+\lambda t)$	$x/u_\delta(x,t)$	$x/(1+\lambda t)$	4
II	$x^M \exp[(1-M)(1+\lambda t)]$	$x/u_\delta(x,t)$	$x\exp(1+\lambda t)$	5
III	$u_\delta=0$ for $t<0$ $u_\delta=U$ for $t\geq 0$	$x\xi/U$	$1-\exp(-Ut/x)$	6

Rayleigh solution valid for small time and the Blasius solution valid for large time.

Since we are interested in investigating the coefficient α_4 and in particular wish to know if and when α_4 vanishes, we will focus our attention on the minimum value of α_4. Since $\alpha_4 = g^2(x,t)[(\partial\xi/\partial t) + u_\delta(\partial\xi/\partial x)f']$, the minimum value of α_4 depends upon the coefficients $g^2(\partial\xi/\partial t)$ and $u_\delta g^2(\partial\xi/\partial x)$ as well as on the local normalized velocity $f'(\xi,\eta)$. In both cases I and II the coefficients $g^2(\partial\xi/\partial t)$ and $g^2 u_\delta(\partial\xi/\partial x)$ are positive (since solutions can only be obtained for positive values of the unsteadiness parameter λ). Then as long as $f'\geq 0$ the minimum vaue of α_4 occurs at the wall ($\eta=0$) where $f'=0$. Once there is a reversal of the flow ($f'<0$) the minimum value occurs at the point where f' has its maximum negative value.

The behavior of the minimum of α_4 for case I with $\lambda=1.5$ is shown in Fig. 1. In this case the minimum value of α_4 first increases with increasing ξ but then reaches a peak and decreases rapidly. The point where the minimum value of α_4 becomes zero corresponds to separation as described by the Moore-Rott-Sears criteion. The variation of the minimum value of α_4 with ξ shown in Fig. 1 is typical of the variations found in each of the solutions of case I. In each solution the point at which α_4 became zero corresponds to unsteady separation.

The behavior of the minimum of α_4 for case II with $\lambda=0.4$ and $M=-0.05$ is also shown in Fig. 1. The minimum value of α_4 first increases with increasing ξ, reaches a peak and then decreases rapidly, just as in case I. This result is typical of the variations found in each of the case II solutions calculated. Again, in each solution the point at which the minimum value of α_4 becomes zero corresponds to separation as described by the Moore-Rott-Sears criterion. The results obtained for both case I and case II flows tend to substantiate the postulate of Wang and Shen.

We now consider case III. The solution for this case describes the development of the viscous flow over a semi-infinite flat plate which is impulsively sent into motion. This is a classical problem which has been studied extensively over the past 25 years.[7-9] The solution to this problem has been difficult to obtain because of the mixed nature of the equations describing the flow. However, Hall[8] obtained a finite difference solution in three independent variables (x,y,t) in 1969 and Dennis[9] obtained a solution in two scaled coordinates in 1972. The solution presented here is that obtained by Williams and Rhyne[6] using a new scaling. The present solution can be shown to be identical to the solution obtained by Dennis by using the appropriate scaling transformation.

In this case the coefficient $g^2(\partial\xi/\partial t)$ is always positive or zero while the coefficient $g^2 u_\delta(\partial\xi/\partial x)$ is always negative or zero. The velocity profile, and hence f', is always positive. Thus, the minimum value of α_4 always occurs at the outer edge of the boundary layer where $f'=1$. For this flow, then, it is possible to calculate the variation of the minimum value of α_4 with ξ without even calculating the solution in detail. The calculated variation of the minimum value of α_4 is also shown in Fig. 1. It is seen that the minimum value of α_4 passes through zero and is negative for a substantial portion of the flowfield. Furthermore, separation is not expected for this flow, and the computational results do not indicate any feature of the flow which would be interpreted as separation. On the basis of this result, we are forced to conclude that the vanishing of the minimum value of α_4 at some interior point in the fluid cannot be taken, by itself, as a "unified separation criteria for both the steady and unsteady cases."

Finally, we note that in case III the vanishing of the minimum value of α_4 has a clear physical significance: it provides an indication of the time at which the presence of the plate leading edge is first felt in the fluid at a given distance downstream of the leading edge.[7] This same interpretation for the significance of $\alpha_4|_{min}=0$ cannot be applied in cases I and II. We are then left with a paradox regarding the significance of $\alpha_4|_{min}=0$. In certain cases $\alpha_4|_{min}=0$ appears to indicate separation, in either steady or unsteady flow, while in other cases $\alpha_4|_{min}=0$ is indicative of the time at which leading edge effects are first felt at a given station. There is one other major difference between these two types of flow. The numerical calculations of flows of cases I or II indicate that as the point at which $\alpha_4|_{min}=0$ is approached (as separation is approached) the vertical component of velocity becomes very large. It is well known, however, that for the flow represented by case III, the vertical velocity is identically zero prior to the point at which $\alpha_4|_{min}=0$. It may be that the appropriate computational indicator of an approach to unsteady (or steady) separation is the simultaneous rapid decrease, towards zero, of the coefficient α_4 and a rapid increase in the vertical component of velocity. This point is presently being investigated further.

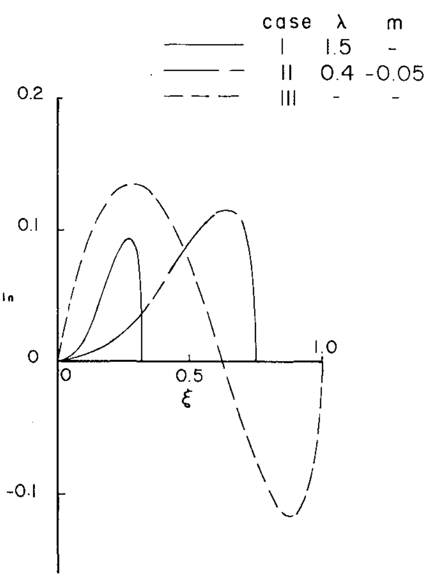

Fig. 1 Variation of the minimum value of α_4 with ξ for three flows.

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Strouhal Number Influence on Flight Effects on Jet Noise Radiated from Convecting Quadrupoles

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Nomenclature

c_f = speed of sound in the simulated flow inducing flight effects
c_0 = speed of sound in the ambient quiescent fluid
I = relative intensity amplification
k_0 = wave number, ω_0/c_0
M_c = convection Mach number, 0.65 M_j
M_f = flight Mach number, U_f/c_0
M_j = jet Mach number, U_j/c_0
r_0 = radius of the jet
St = Strouhal number, $\omega_0 r_0/U_j$
θ = angle between the directions of convection and emission at the retarded time
ρ_f = density of the simulated flow inducing flight effects ($\sim\rho_0$)
ρ_j = density of the jet
ω_0 = source frequency

Introduction

THIS work is a complementary extension of our recent work[1,2] which discovers several interesting features recognizable in experimental jet noise fields as well as in actual flyovers. However, those predictions are descriptive of a low frequency situation and, consequently, we report here a complementary extension to include the high frequency features which will be reflected in our discussion on the higher Strouhal number influence on the flight effects. From our knowledge of the distant pressure field, developed in Eq. (13) of Ref. 2, one can easily write an expression for relative intensity amplification factor, I, as

$$I = \left| \frac{\psi(M_f)}{\psi(M_f=0)} \right|^2 \left[\frac{1-(c_o/c_f)M_c\cos\theta}{1-(c_o/c_f)(M_c-M_f)\cos\theta} \right]^6 \qquad (1)$$

This, however, is the ratio of the intensity of noise in flight to that without flight. As the dependency of ψ rests on M_f, M_j, St, and θ, it will be interesting to plot graphs in Figs. 1-3 for 10 $\log_{10} I$ against θ as the independent variable, with M_f, M_j, and St as the fixed variables; it has to be pointed out that while handling the above equation, $(kr)_o$ in ψ is replaced by $St\ M_j$, to which it is directly related.

The Strouhal number values chosen are $St = 0.5$, 1.0, and 3.0, respectively in Figs. 1-3. The computation is facilitated by expressing the Bessel functions through their equivalent Chebyshev series.[3]

Discussion of Graphs

Figures 1-3 describe the change in directional distribution of relative intensity amplification which arises as a result of radiation from an axial point quadrupole convecting along the jet centerline at Mach number M_c by a jet flow of Mach number M_j under the influence of flight at Mach number M_f, the values of which are indicated against the curves. The angle θ is measured from the direction of convection to the direction of emission at the retarded time.

Like the low frequency case reported in our earlier work, the high frequency case of our present work shows many of the important features of the flight effects on noise which are in common agreement in both the situations. As is obvious from the plots in Figs. 1-3, these common features are:

1) Forward arc amplification is caused as a result of forward speed.

2) Flight effects steadily amplify noise in the forward quadrant ($\pi/2<\theta\leq\pi$) and diminish noise in the aft quadrant ($0\leq\theta<\pi/2$).

3) Amplification in the forward quadrant decreases when jet velocity is increased (up to its critical vlaue) with, however, a weaker attenuation in the aft quadrant.

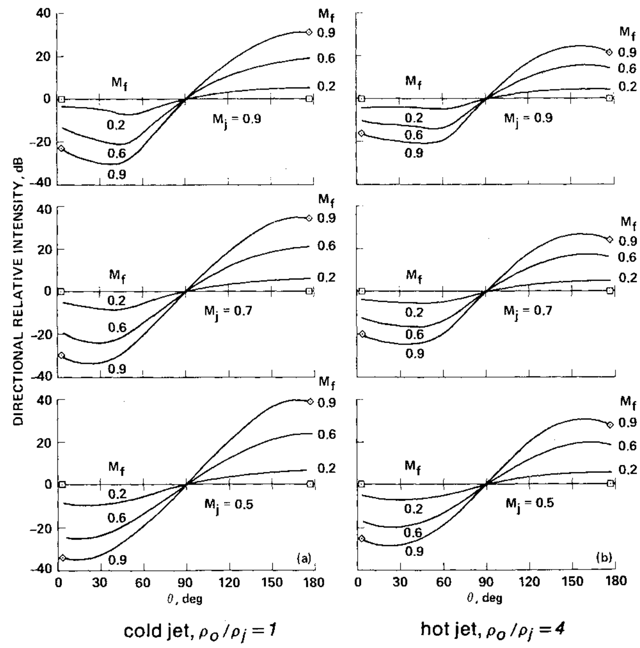

Fig. 1 Variation in directional distribution of relative intensity *I* showing the forward arc amplification cum rear arc attenuation, *St* = 0.5.

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